Variability Abstraction and Refinement for Game-based Lifted Model Checking of full CTL

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Abstract. Variability models allow effective building of many custom model variants for various configurations. Lifted model checking for a variability model is capable of verifying all its variants simultaneously in a single run by exploiting the similarities between the variants. The computational cost of lifted model checking still greatly depends on the number of variants (the size of configuration space), which is often huge. One of the most promising approaches to fighting the configuration space explosion problem in lifted model checking are *variability abstractions*. In this work, we define a novel game-based approach for variability-specific abstraction and refinement for lifted model checking of the full CTL, interpreted over 3-valued semantics. We propose a direct algorithm for solving a 3-valued (abstract) lifted model checking game. In case the result of model checking an abstract variability model is indefinite, we suggest a new notion of refinement, which eliminates indefinite results. This provides an iterative incremental variability-specific abstraction and refinement framework, where refinement is applied only where indefinite results exist and definite results from previous iterations are reused.

1 Introduction

Software Product Line (SPL) [6] is an efficient method for systematic development of a family of related models, known as *variants* (*valid products*), from a common code base. Each variant is specified in terms of *features* (static configuration options) selected for that particular variant. SPLs are particulary popular in the embedded and critical system domains (e.g. cars, phones, avionics, healthcare).

Lifted model checking [4,5] is a useful approach for verifying properties of variability models (SPLs). Given a variability model and a specification, the lifted model checking algorithm, unlike the standard non-lifted one, returns precise conclusive results for all individual variants, that is, for each variant it reports whether it satisfies or violates the specification. The main disadvantage of lifted model checking is the *configuration space explosion problem*, which refers to the high number of variants in the variability model. In fact, exponentially many variants can be derived from only few configuration options (features). One of the most successful approaches to fighting the configuration space explosion are so-called *variability abstractions* [14,15,17,12]. They hide some of the configuration details, so that many of the concrete configurations become indistinguishable and can be collapsed into a single abstract configuration (variant). This results in smaller abstract variability models with a smaller number of abstract configurations. In order to be conservative w.r.t. the full CTL temporal logic, abstract variability models have two types of transitions: *may-transitions* which represent possible transitions in the concrete model, and *must-transitions* which represent the definite transitions in the concrete model. May and must transitions correspond to over and under approximations, and are needed in order to preserve universal and existential CTL properties, respectively.

Here we consider the 3-valued semantics for interpreting CTL formulae over abstract variability models. This semantics evaluates a formula on an abstract model to either *true*, *false*, or *indefinite*. Abstract variability models are designed to be conservative for both *true* and *false*. However, the *indefinite* answer gives no information on the value of the formula on the concrete model. In this case, a refinement is needed in order to make the abstract models more precise.

The technique proposed here significantly extends the scope of existing automatic variability-specific abstraction refinement procedures [8,18], which currently support the verification of universal LTL properties only. They use conservative variability abstractions to construct over-approximated abstract variability models, which preserve LTL properties. If a spurious counterexample (introduced due to the abstraction) is found in the abstract model, the procedures [8,18] use Craig interpolation to extract relevant information from it in order to define the refinement of abstract models. Variability abstractions that preserve all (universal and existential) CTL properties have been previously introduced [12], but without an automatic mechanism for constructing them and no notion of refinement. The abstractions [12] has to be constructed manually by an engineer before verification. In order to make the entire verification procedure automatic, we need to develop an abstraction and refinement framework for CTL properties.

In this work, we propose the first variability-specific abstraction refinement procedure for automatically verifying arbitrary formulae of CTL. To achieve this aim, model checking *games* [26,24,25] represent the most suitable framework for defining the refinement. In this way, we establish a brand new connection between games and family-based (SPL) model checking. The refinement is defined by finding the reason for the indefinite result of an algorithm that solves the corresponding model checking game, which is played by two players: Player $\forall$ (trying to refute the formula Φ on an abstract model $\mathcal{M}$) and Player $\exists$ (trying to verify Φ on $\mathcal{M}$). The game is played on a *game board*, which consists of configurations of the form (s, Φ') where s is a state of the abstract model $\mathcal{M}$ and Φ' is a subformula of Φ, such that the value of Φ' in s is relevant for determining the final model checking result. The players make moves between configurations in which they try to verify or refute Φ' in s. All possible plays of a game are captured in the game-graph, whose nodes are the elements of the game board and whose edges are the possible moves of the players. The model checking game is solved via a coloring algorithm which colors each node (s, Φ') in the game-graph by T, F, or ? iff the value of Φ' in s is *true*, *false*, or indefinite, respectively.

Player $\forall$ has a winning strategy at the node (s, Φ') iff the node is colored by F iff Φ' does not hold in s, and Player $\exists$ has a winning strategy at (s, Φ') iff the node is colored by T iff Φ' holds in s. In addition, it is also possible that neither of players has a winning strategy, in which case the node is colored by ? and the value of Φ' in s is indefinite. In this case, we want to refine the abstract model. We can find the reason for the tie by examining the game-graph. We choose a refinement criterion, which splits abstract configurations so that the new, refined abstract configurations represent smaller subsets of concrete configurations.

2 Background

Variability Models. Let $\mathbb{F} = \{A_1, \ldots, A_n\}$ be a finite set of Boolean variables representing the features available in a variability model. A specific subset of features, $k \subseteq \mathbb{F}$, known as *configuration*, specifies a *variant* (valid product) of a variability model. We assume that only a subset $\mathbb{K} \subseteq 2^{\mathbb{F}}$ of configurations are *valid*. An alternative representation of configurations is based upon propositional formulae. Each configuration $k \in \mathbb{K}$ can be represented by a formula: $k(A_1) \wedge \ldots \wedge k(A_n)$, where $k(A_i) = A_i$ if $A_i \in k$, and $k(A_i) = \neg A_i$ if $A_i \notin k$ for $1 \leq i \leq n$.

We use *transition systems* (TS) to describe behaviors of single-systems.

Definition 1. *A transition system (TS) is a tuple* $\mathcal{T} = (S, \mathit{Act}, \mathit{trans}, I, \mathit{AP}, L)$, *where* S *is a set of states;* Act *is a set of actions;* $\mathit{trans} \subseteq S \times \mathit{Act} \times S$ *is a transition relation which is* total*, so that for each state there is an outgoing transition;* $I \subseteq S$ *is a set of initial states;* AP *is a set of atomic propositions; and* $L : S \to 2^{\mathit{AP}}$ *is a labelling function specifying which propositions hold in a state. We write* $s_1 \xrightarrow{\lambda} s_2$ *whenever* $(s_1, \lambda, s_2) \in \mathit{trans}$.

An *execution* (behaviour) of a TS $\mathcal{T}$ is an *infinite* sequence $\rho = s_0\lambda_1 s_1 \lambda_2 \ldots$ with $s_0 \in I$ such that $s_i \xrightarrow{\lambda_{i+1}} s_{i+1}$ for all $i \geq 0$. The *semantics* of the TS $\mathcal{T}$, denoted as $[\![\mathcal{T}]\!]_{TS}$, is the set of its executions.

A *featured transition system* (FTS) is a particular instance of a variability model, which describes the behavior of a whole family of systems in a single monolithic description, where the transitions are guarded by a *presence condition* that identifies the variants they belong to. The presence conditions ψ are drawn from the set of feature expressions, $\mathit{FeatExp}(\mathbb{F})$, which are propositional logic formulae over $\mathbb{F}$: $\psi ::= \mathit{true} \mid A \in \mathbb{F} \mid \neg\psi \mid \psi_1 \wedge \psi_2$. We write $[\![\psi]\!]$ to denote the set of configurations from $\mathbb{K}$ that satisfy ψ, i.e. $k \in [\![\psi]\!]$ iff $k \models \psi$.

Definition 2. *A featured transition system (FTS) represents a tuple* $\mathcal{F} = (S, \mathit{Act}, \mathit{trans}, I, \mathit{AP}, L, \mathbb{F}, \mathbb{K}, \delta)$, *where* $S, \mathit{Act}, \mathit{trans}, I, \mathit{AP}$, *and* L *form a TS;* $\mathbb{F}$ *is the set of available features;* $\mathbb{K}$ *is a set of valid configurations; and* $\delta : \mathit{trans} \to \mathit{FeatExp}(\mathbb{F})$ *is a total function decorating transitions with presence conditions.*

The *projection* of an FTS $\mathcal{F}$ to a configuration $k \in \mathbb{K}$, denoted as $\pi_k(\mathcal{F})$, is the TS $(S, \mathit{Act}, \mathit{trans}', I, \mathit{AP}, L)$, where $\mathit{trans}' = \{t \in \mathit{trans} \mid k \models \delta(t)\}$. We lift the definition of *projection* to sets of configurations $\mathbb{K}' \subseteq \mathbb{K}$, denoted as $\pi_{\mathbb{K}'}(\mathcal{F})$, by keeping the transitions admitted by at least one of the configurations in $\mathbb{K}'$. That

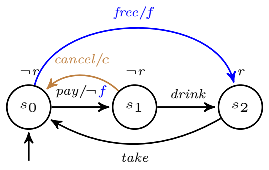


Fig. 1: VendMach

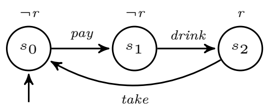


Fig. 2: $\pi_\emptyset$(VendMach)

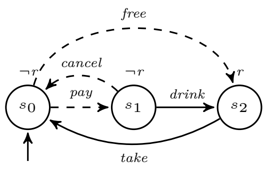


Fig. 3: $\boldsymbol{\alpha}^{\text{join}}$(VendMach)

is, $\pi_{\mathbb{K}'}(\mathcal{F})$, is the FTS $(S, Act, trans', I, AP, L, \mathbb{F}, \mathbb{K}', \delta')$, where $trans' = \{t \in trans \mid \exists k \in \mathbb{K}'.k \models \delta(t)\}$ and $\delta' = \delta|_{trans'}$ is the restriction of δ to $trans'$. The *semantics* of an FTS $\mathcal{F}$, denoted as $[\![\mathcal{F}]\!]_{FTS}$, is the union of behaviours of the projections on all valid variants $k \in \mathbb{K}$, i.e. $[\![\mathcal{F}]\!]_{FTS} = \cup_{k\in\mathbb{K}}[\![\pi_k(\mathcal{F})]\!]_{TS}$.

Modal transition systems (MTSs) [22] are a generalization of transition systems equipped with two transition relations: *must* and *may*. The former (must) is used to specify the required behavior, while the latter (may) to specify the allowed behavior of a system. We will use MTSs for representing abstractions of FTSs.

Definition 3. *A modal transition system (MTS) is represented by a tuple $\mathcal{M} = (S, Act, trans^{may}, trans^{must}, I, AP, L)$, where $trans^{may} \subseteq S \times Act \times S$ describe may transitions of $\mathcal{M}$; $trans^{must} \subseteq S \times Act \times S$ describe must transitions of $\mathcal{M}$, such that $trans^{may}$ is total and $trans^{must} \subseteq trans^{may}$.*

A *may-execution* in $\mathcal{M}$ is an execution (infinite sequence) with all its transitions in $trans^{\text{may}}$; whereas a *must-execution* in $\mathcal{M}$ is a maximal sequence with all its transitions in $trans^{\text{must}}$, which cannot be extended with any other transition from $trans^{\text{must}}$. Note that since $trans^{\text{must}}$ is not necessarily total, must-executions can be finite. We use $[\![\mathcal{M}]\!]^{\text{may}}_{MTS}$ (resp., $[\![\mathcal{M}]\!]^{\text{must}}_{MTS}$) to denote the set of all may-executions (resp., must-executions) in $\mathcal{M}$ starting in an initial state.

Example 1. Throughout this paper, we will use a beverage vending machine as a running example [4]. Figure 1 shows the FTS of a VendMach family. It has two features, and each of them is assigned an identifying letter and a color. The features are: `CancelPurchase` (c, in brown), for canceling a purchase after a coin is entered; and `FreeDrinks` (f, in blue) for offering free drinks. Each transition is labeled by an *action* followed by a *feature expression*. For instance, the transition $s_0 \xrightarrow{free/f} s_2$ is included in variants where the feature f is enabled. For clarity, we omit to write the presence condition *true* in transitions. There is only one atomic proposition `served` $\in AP$, which is abbreviated as r. Note that $r \in L(s_2)$, whereas $r \notin L(s_0)$ and $r \notin L(s_1)$.

By combining various features, a number of variants of this VendMach can be obtained. The set of valid configurations is: $\mathbb{K}^{\text{VM}} = \{\emptyset, \{c\}, \{f\}, \{c, f\}\}$ (or, equivalently $\mathbb{K}^{\text{VM}} = \{\neg c \wedge \neg f, c \wedge \neg f, \neg c \wedge f, c \wedge f\}$). Figure 2 shows a basic version of VendMach that only serves a drink, described by the configuration: $\emptyset$ (or, as formula $\neg c \wedge \neg f$). It takes a coin, serves a drink, opens a compartment so the customer can take the drink. Figure 3 shows an MTS, where must transitions are denoted by solid lines, while may transitions by dashed lines. □

CTL Properties. We present Computation Tree Logic (CTL) [1] for specifying system properties. CTL state formulae Φ are given by:

$$\Phi ::= true \mid false \mid l \mid \Phi_1 \wedge \Phi_2 \mid \Phi_1 \vee \Phi_2 \mid A\phi \mid E\phi, \qquad \phi ::= \bigcirc\Phi \mid \Phi_1 \mathsf{U} \Phi_2 \mid \Phi_1 \mathsf{V} \Phi_2$$

where $l \in Lit = AP \cup \{\neg a \mid a \in AP\}$ and ϕ represent CTL path formulae. Note that the CTL state formulae Φ are given in negation normal form ($\neg$ is applied only to atomic propositions). The path formula $\bigcirc\Phi$ can be read as "in the next state Φ", $\Phi_1 \mathsf{U} \Phi_2$ can be read as "Φ_1 until Φ_2", and its dual $\Phi_1 \mathsf{V} \Phi_2$ can be read as "Φ_2 while not Φ_1" (where Φ_1 may never hold).

We assume the standard CTL semantics over TSs is given [1] (see also [16, Appendix A]). We write $[\mathcal{T} \models \Phi] = tt$ to denote that $\mathcal{T}$ satisfies the formula Φ, whereas $[\mathcal{T} \models \Phi] = ff$ to denote that $\mathcal{T}$ does not satisfy Φ.

We say that an FTS $\mathcal{F}$ satisfies a CTL formula Φ, written $[\mathcal{F} \models \Phi] = tt$, iff all its valid variants satisfy the formula, i.e. $\forall k \in \mathbb{K}. [\pi_k(\mathcal{F}) \models \Phi] = tt$. Otherwise, we say $\mathcal{F}$ does not satisfy Φ, written $[\mathcal{F} \models \Phi] = ff$. In this case, we also want to determine a non-empty set of violating variants $\mathbb{K}' \subseteq \mathbb{K}$, such that $\forall k' \in \mathbb{K}'. [\pi_{k'}(\mathcal{F}) \models \Phi] = ff$ and $\forall k \in \mathbb{K} \backslash \mathbb{K}'. [\pi_k(\mathcal{F}) \models \Phi] = tt$.

We define the 3-valued semantics of CTL over an MTS $\mathcal{M}$ slightly differently from the semantics for TSs. A CTL state formula Φ is satisfied in a state s of an MTS $\mathcal{M}$, denoted $[\mathcal{M}, s \models^3 \Phi]$, iff ($\mathcal{M}$ is omitted when clear from context): [4]

$$\textbf{(1)}\ \ [s \models^3 a] = \begin{cases} tt, & \text{if } a \in L(s) \\ ff, & \text{if } a \notin L(s) \end{cases}, \quad [s \models^3 \neg a] = \begin{cases} tt, & \text{if } a \notin L(s) \\ ff, & \text{if } a \in L(s) \end{cases}$$

$$\textbf{(2)}\ \ [s \models^3 \Phi_1 \wedge \Phi_2] = \begin{cases} tt, & \text{if } [s \models^3 \Phi_1] = tt \text{ and } [s \models^3 \Phi_2] = tt \\ ff, & \text{if } [s \models^3 \Phi_1] = ff \text{ or } [s \models^3 \Phi_2] = ff \\ \bot, & \text{otherwise} \end{cases}$$

$$\textbf{(3)}\ \ [s \models^3 A\phi] = \begin{cases} tt, & \text{if } \forall \rho \in [\![\mathcal{M}]\!]^{\text{may},s}_{MTS}. [\rho \models^3 \phi] = tt \\ ff, & \text{if } \exists \rho \in [\![\mathcal{M}]\!]^{\text{must},s}_{MTS}. [\rho \models^3 \phi] = ff \\ \bot, & \text{otherwise} \end{cases}$$

$$[s \models^3 E\phi] = \begin{cases} tt, & \text{if } \exists \rho \in [\![\mathcal{M}]\!]^{\text{must},s}_{MTS}. [\rho \models^3 \phi] = tt \\ ff, & \text{if } \forall \rho \in [\![\mathcal{M}]\!]^{\text{may},s}_{MTS}. [\rho \models^3 \phi] = ff \\ \bot, & \text{otherwise} \end{cases}$$

where $[\![\mathcal{M}]\!]^{\text{may},s}_{MTS}$ (resp., $[\![\mathcal{M}]\!]^{\text{must},s}_{MTS}$) denotes the set of all may-executions (must-executions) starting in the state s of $\mathcal{M}$. Satisfaction of a path formula ϕ for a may- or must-execution $\rho = s_0\lambda_1 s_1 \lambda_2 \ldots$ of an MTS $\mathcal{M}$ (we write $\rho_i = s_i$ to denote the i-th state of ρ, and $|\rho|$ to denote the number of states in ρ), denoted $[\mathcal{M}, \rho \models^3 \phi]$, is defined as ($\mathcal{M}$ is omitted when clear from context):

$$\textbf{(4)}\ \ [\rho \models^3 (\Phi_1 \mathsf{U} \Phi_2)] = \begin{cases} tt, & \text{if } \exists 0 \leq i \leq |\rho|. \big([\rho_i \models^3 \Phi_2] = tt \wedge (\forall j < i. [\rho_j \models^3 \Phi_1] = tt)\big) \\ ff, & \text{if } \begin{array}{l} \forall 0 \leq i \leq |\rho|. \big(\forall j < i. [\rho_j \models^3 \Phi_1] \neq ff \Longrightarrow [\rho_i \models^3 \Phi_2] = ff\big) \\ \wedge\ \forall i \geq 0. [\rho_i \models^3 \Phi_1] \neq ff \Longrightarrow |\rho| = \infty \end{array} \\ \bot, & \text{otherwise} \end{cases}$$

[4] See [16, Appendix A] for definitions of $[s \models^3 \Phi_1 \vee \Phi_2]$, $[\rho \models^3 \bigcirc\Phi]$, and $[\rho \models^3 (\Phi_1 \mathsf{V} \Phi_2)]$.

A MTS $\mathcal{M}$ satisfies a formula Φ, written $[\mathcal{M} \models^3 \Phi] = tt$, iff $\forall s_0 \in I. [s_0 \models^3 \Phi] = tt$. We say that $[\mathcal{M} \models^3 \Phi] = \mathit{ff}$ if $\exists s_0 \in I. [s_0 \models^3 \Phi] = \mathit{ff}$. Otherwise, $[\mathcal{M} \models^3 \Phi] = \bot$.

Example 2. Consider the FTS VendMach and MTS $\boldsymbol{\alpha}^{\text{join}}(\text{VendMach})$ in Figures 1 and 3. The property $\Phi_1 = A(\neg r \mathsf{U} r)$ states that in the initial state along every execution will eventually reach the state where r holds. Note that $[\text{VendMach} \models \Phi_1] = \mathit{ff}$. E.g., if the feature c is enabled, a counter-example where the state s_2 that satisfies r is never reached is: $s_0 \to s_1 \to s_0 \to \ldots$. The set of violating products is $[\![c]\!] = \{\{c\}, \{f, c\}\} \subseteq \mathbb{K}^{VM}$. However, $[\pi_{[\![\neg c]\!]}(\text{VendMach}) \models \Phi_1] = tt$. We also have that $[\boldsymbol{\alpha}^{\text{join}}(\text{VendMach}) \models^3 \Phi_1] = \bot$, since (1) there is a may-execution in $\boldsymbol{\alpha}^{\text{join}}(\text{VendMach})$ where s_2 is never reached: $s_0 \to s_1 \to s_0 \to \ldots$, and (2) there is no must-execution that violates Φ_1.

Consider the property $\Phi_2 = E(\neg r \mathsf{U} r)$, which describes a situation where in the initial state there exists an execution that will eventually reach s_2 that satisfies r. Note that $[\text{VendMach} \models \Phi_2] = tt$, since even for variants with the feature c there is a continuation from the state s_1 to s_2. But, $[\boldsymbol{\alpha}^{\text{join}}(\text{VendMach}) \models \Phi_2] = \bot$ since (1) there is no a must-execution in $\boldsymbol{\alpha}^{\text{join}}(\text{VendMach})$ that reaches s_2 from s_0, and (2) there is a may-execution that satisfies Φ_2. □

3 Abstraction of FTSs

We now introduce the variability abstractions [12] which preserve full CTL. We start working with Galois connections[5] between Boolean complete lattices of feature expressions, and then induce a notion of abstraction of FTSs.

The Boolean complete lattice of feature expressions (propositional formulae over $\mathbb{F}$) is: $(\mathit{FeatExp}(\mathbb{F})_{/\equiv}, \models, \vee, \wedge, \mathit{true}, \mathit{false}, \neg)$. The elements of the domain $\mathit{FeatExp}(\mathbb{F})_{/\equiv}$ are equivalence classes of propositional formulae $\psi \in \mathit{FeatExp}(\mathbb{F})$ obtained by quotienting by the semantic equivalence $\equiv$. The ordering $\models$ is the standard entailment between propositional logics formulae, whereas the least upper bound and the greatest lower bound are just logical disjunction and conjunction respectively. Finally, the constant *false* is the least, *true* is the greatest element, and negation is the complement operator.

Over-approximating abstractions. The *join abstraction*, $\boldsymbol{\alpha}^{\text{join}}$, replaces each feature expression ψ with *true* if there exists at least one configuration from $\mathbb{K}$ that satisfies ψ. The abstract set of features is empty: $\boldsymbol{\alpha}^{\text{join}}(\mathbb{F}) = \emptyset$, and abstract set of configurations is a singleton: $\boldsymbol{\alpha}^{\text{join}}(\mathbb{K}) = \{\mathit{true}\}$. The abstraction and concretization functions between $\mathit{FeatExp}(\mathbb{F})$ and $\mathit{FeatExp}(\emptyset)$ are:

$$\boldsymbol{\alpha}^{\text{join}}(\psi) = \begin{cases} \mathit{true} & \text{if } \exists k \in \mathbb{K}. k \models \psi \\ \mathit{false} & \text{otherwise} \end{cases} \qquad \boldsymbol{\gamma}^{\text{join}}(\psi) = \begin{cases} \mathit{true} & \text{if } \psi \text{ is } \mathit{true} \\ \bigvee_{k \in 2^{\mathbb{F}} \setminus \mathbb{K}} k & \text{if } \psi \text{ is } \mathit{false} \end{cases}$$

which form a Galois connection [15]. In this way, we obtain a single abstract variant that includes all transitions occurring in any variant.

[5] $\langle L, \leq_L \rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle M, \leq_M \rangle$ is a *Galois connection* between complete lattices L (concrete domain) and M (abstract domain) iff $\alpha : L \to M$ and $\gamma : M \to L$ are total functions that satisfy: $\alpha(l) \leq_M m \iff l \leq_L \gamma(m)$, for all $l \in L, m \in M$.

Under-approximating abstractions. The *dual join abstraction*, $\widetilde{\boldsymbol{\alpha}^{\text{join}}}$, replaces each feature expression ψ with *true* if all configurations from $\mathbb{K}$ satisfy ψ. The abstraction and concretization functions between $FeatExp(\mathbb{F})$ and $FeatExp(\emptyset)$, forming a Galois connection [12], are defined as [9]: $\widetilde{\boldsymbol{\alpha}^{\text{join}}} = \neg \circ \boldsymbol{\alpha}^{\text{join}} \circ \neg$ and $\widetilde{\boldsymbol{\gamma}^{\text{join}}} = \neg \circ \boldsymbol{\gamma}^{\text{join}} \circ \neg$, that is:

$$\widetilde{\boldsymbol{\alpha}^{\text{join}}}(\psi) = \begin{cases} \mathit{true} & \text{if } \forall k \in \mathbb{K}. k \models \psi \\ \mathit{false} & \text{otherwise} \end{cases} \qquad \widetilde{\boldsymbol{\gamma}^{\text{join}}}(\psi) = \begin{cases} \bigwedge_{k \in 2^{\mathbb{F}} \setminus \mathbb{K}} (\neg k) & \text{if } \psi \text{ is } \mathit{true} \\ \mathit{false} & \text{if } \psi \text{ is } \mathit{false} \end{cases}$$

In this way, we obtain a single abstract variant that includes only those transitions that occur in all variants.

Abstract MTS and Preservation of CTL. Given a Galois connection $(\boldsymbol{\alpha}^{\text{join}}, \boldsymbol{\gamma}^{\text{join}})$ defined on the level of feature expressions, we now define the abstraction of an FTS as an MTS with two transition relations: one (may) preserving universal properties, and the other (must) preserving existential properties. The may transitions describe the behaviour that is possible in some variant of the concrete FTS, but not need be realized in the other variants; whereas the must transitions describe behaviour that has to be present in all variants of the FTS.

Definition 4. *Given the FTS $\mathcal{F} = (S, Act, trans, I, AP, L, \mathbb{F}, \mathbb{K}, \delta)$, define MTS $\boldsymbol{\alpha}^{\text{join}}(\mathcal{F}) = (S, Act, trans^{may}, trans^{must}, I, AP, L)$ to be its* abstraction*, where $trans^{may} = \{t \in trans \mid \boldsymbol{\alpha}^{\text{join}}(\delta(t)) = \mathit{true}\}$, and $trans^{must} = \{t \in trans \mid \widetilde{\boldsymbol{\alpha}^{\text{join}}}(\delta(t)) = \mathit{true}\}$.*

Note that the abstract model $\boldsymbol{\alpha}^{\text{join}}(\mathcal{F})$ has no variability in it, i.e. it contains only one abstract configuration. We now show that the 3-valued semantics of the MTS $\boldsymbol{\alpha}^{\text{join}}(\mathcal{F})$ is designed to be *sound* in the sense that it preserves both satisfaction (*tt*) and refutation (*ff*) of a formula from the abstract model to the concrete one. However, if the truth value of a formula in the abstract model is $\bot$, then its value over the concrete model is not known. We prove [16, Appendix B]:

Theorem 1 (Preservation results). *For every $\Phi \in CTL$, we have:*

(1) $[\boldsymbol{\alpha}^{\text{join}}(\mathcal{F}) \models^3 \Phi] = tt \implies [\mathcal{F} \models \Phi] = tt$.
(2) $[\boldsymbol{\alpha}^{\text{join}}(\mathcal{F}) \models^3 \Phi] = ff \implies [\mathcal{F} \models \Phi] = ff$ *and* $[\pi_k(\mathcal{F}) \models \Phi] = ff$ *for all* $k \in \mathbb{K}$.

Divide-and-conquer strategy. The problem of evaluating $[\mathcal{F} \models \Phi]$ can be reduced to a number of smaller problems by partitioning the configuration space $\mathbb{K}$. Let the subsets $\mathbb{K}_1, \mathbb{K}_2, \ldots, \mathbb{K}_n$ form a *partition* of the set $\mathbb{K}$. Then, $[\mathcal{F} \models \Phi] = tt$ iff $[\pi_{\mathbb{K}_i}(\mathcal{F}) \models \Phi] = tt$ for all $i = 1, \ldots, n$. Also, $[\mathcal{F} \models \Phi] = ff$ iff $[\pi_{\mathbb{K}_j}(\mathcal{F}) \models \Phi] = ff$ for some $1 \leq j \leq n$. By using Theorem 1, we obtain the following result.

Corollary 1. *Let $\mathbb{K}_1, \mathbb{K}_2, \ldots, \mathbb{K}_n$ form a* partition *of $\mathbb{K}$.*

(1) *If $[\boldsymbol{\alpha}^{\text{join}}(\pi_{\mathbb{K}_1}(\mathcal{F})) \models \Phi] = tt \wedge \ldots \wedge [\boldsymbol{\alpha}^{\text{join}}(\pi_{\mathbb{K}_n}(\mathcal{F})) \models \Phi] = tt$, then $[\mathcal{F} \models \Phi] = tt$.*
(2) *If $[\boldsymbol{\alpha}^{\text{join}}(\pi_{\mathbb{K}_j}(\mathcal{F})) \models \Phi] = ff$ for some $1 \leq j \leq n$, then $[\mathcal{F} \models \Phi] = ff$ and $[\pi_k(\mathcal{F}) \models \Phi] = ff$ for all $k \in \mathbb{K}_j$.*

Example 3. Recall the FTS VendMach of Fig. 1. Figure 3 shows the MTS $\boldsymbol{\alpha}^{\text{join}}(\textsc{VendMach})$, where the allowed (may) part of the behavior includes the transitions that are associated with the optional features c and f in VendMach, and the required (must) part includes transitions with the presence condition *true*. Consider the properties introduced in Example 2. We have $[\boldsymbol{\alpha}^{\text{join}}(\textsc{VendMach}) \models^3 \Phi_1] = \bot$ and $[\boldsymbol{\alpha}^{\text{join}}(\textsc{VendMach}) \models^3 \Phi_2] = \bot$, so we cannot conclude whether Φ_1 and Φ_2 are satisfied by VendMach or not. $\square$

4 Game-based Abstract Lifted Model Checking

The 3-valued model checking game [24,25] on an MTS $\mathcal{M}$ with state set S, a state $s \in S$, and a CTL formula Φ is played by Player $\forall$ and Player $\exists$ in order to evaluate Φ in s of $\mathcal{M}$. The goal of Player $\forall$ is either to refute Φ on $\mathcal{M}$ or to prevent Player $\exists$ from verifying it. The goal of Player $\exists$ is either to verify Φ on $\mathcal{M}$ or to prevent Player $\forall$ from refuting it. The *game board* is the Cartesian product $S \times sub(\Phi)$, where $sub(\Phi)$ is defined as:

$$\begin{array}{l} \text{if } \Phi = true, false, l, \text{then } sub(\Phi) = \{\Phi\}; \text{ if } \Phi = \text{Æ}\bigcirc\Phi_1, \text{then } sub(\Phi) = \{\Phi\} \cup sub(\Phi_1) \\ \text{if } \Phi = \Phi_1 \wedge \Phi_2, \Phi_1 \vee \Phi_2, \text{ then } sub(\Phi) = \{\Phi\} \cup sub(\Phi_1) \cup sub(\Phi_2) \\ \text{if } \Phi = \text{Æ}(\Phi_1 \mathsf{U} \Phi_2), \text{Æ}(\Phi_1 \mathsf{V} \Phi_2), \text{ then } sub(\Phi) = exp(\Phi) \cup sub(\Phi_1) \cup sub(\Phi_2) \end{array}$$

where Æ ranges over both A and E. The expansion $exp(\Phi)$ is defined as:

$$\begin{array}{l} \Phi = \text{Æ}(\Phi_1 \mathsf{U} \Phi_2) : exp(\Phi) = \{\Phi, \Phi_2 \vee (\Phi_1 \wedge \text{Æ} \bigcirc \Phi), \Phi_1 \wedge \text{Æ} \bigcirc \Phi, \text{Æ} \bigcirc \Phi\} \\ \Phi = \text{Æ}(\Phi_1 \mathsf{V} \Phi_2) : exp(\Phi) = \{\Phi, \Phi_2 \wedge (\Phi_1 \vee \text{Æ} \bigcirc \Phi), \Phi_1 \vee \text{Æ} \bigcirc \Phi, \text{Æ} \bigcirc \Phi\} \end{array}$$

A *single play* from (s, Φ) is a possibly infinite sequence of configurations $C_0 \to_{p_0} C_1 \to_{p_1} C_2 \to_{p_2} \ldots$, where $C_0 = (s, \Phi)$, $C_i \in S \times sub(\Phi)$, and $p_i \in \{\text{Player } \forall, \text{Player } \exists\}$. The subformula in C_i determines which player p_i makes the next move. The possible moves at each configuration are:

(1) $C_i = (s, false)$, $C_i = (s, true)$, $C_i = (s, l)$: the play is finished. Such configurations are called *terminal*.
(2) if $C_i = (s, A \bigcirc \Phi)$, Player $\forall$ chooses a must-transition $s \to s'$ (for refutation) or a may-transition $s \to s'$ of $\mathcal{M}$ (to prevent satisfaction), and $C_{i+1} = (s', \Phi)$.
(3) if $C_i = (s, E \bigcirc \Phi)$, Player $\exists$ chooses a must-transition $s \to s'$ (for satisfaction) or a may-transition $s \to s'$ of $\mathcal{M}$ (to prevent refutation), and $C_{i+1} = (s', \Phi)$.
(4) if $C_i = (s, \Phi_1 \wedge \Phi_2)$, then Player $\forall$ chooses $j \in \{1, 2\}$ and $C_{i+1} = (s, \Phi_j)$.
(5) if $C_i = (s, \Phi_1 \vee \Phi_2)$, then Player $\exists$ chooses $j \in \{1, 2\}$ and $C_{i+1} = (s, \Phi_j)$.
(6),(7) if $C_i = (s, \text{Æ}(\Phi_1 \mathsf{U} \Phi_2))$, then $C_{i+1} = (s, \Phi_2 \vee (\Phi_1 \wedge \text{Æ} \bigcirc \text{Æ}(\Phi_1 \mathsf{U} \Phi_2)))$.
(8),(9) if $C_i = (s, \text{Æ}(\Phi_1 \mathsf{V} \Phi_2))$, then $C_{i+1} = (s, \Phi_2 \wedge (\Phi_1 \vee \text{Æ} \bigcirc \text{Æ}(\Phi_1 \mathsf{V} \Phi_2)))$.

The moves $(6) - (9)$ are deterministic, thus any player can make them.

A play is a *maximal play* iff it is infinite or ends in a terminal configuration. A play is infinite [26] iff there is exactly one subformula of the form $A\mathsf{U}$, $A\mathsf{V}$, $E\mathsf{U}$, or $E\mathsf{V}$ that occurs infinitely often in the play. Such a subformula is called a *witness*. We have the following *winning criteria*:

- Player $\forall$ *wins* a (maximal) play iff in each configuration of the form $C_i = (s, A \bigcirc \Phi)$, Player $\forall$ chooses a move based on must-transitions and one of the following holds: (1) the play is finite and ends in a terminal configuration of the form $C_i = (s, \mathit{false})$ or $C_i = (s, a)$ where $a \notin L(s)$ or $C_i = (s, \neg a)$ where $a \in L(s)$; (2) the play is infinite and the witness is of the form $A\mathsf{U}$ or $E\mathsf{U}$.
- Player $\exists$ *wins* a (maximal) play iff in each configuration of the form $C_i = (s, E \bigcirc \Phi)$, Player $\exists$ chooses a move based on must-transitions and one of the following holds: (1) the play is finite and ends in a terminal configuration of the form $C_i = (s, \mathit{true})$ or $C_i = (s, a)$ where $a \in L(s)$ or $C_i = (s, \neg a)$ where $a \notin L(s)$; (2) the play is infinite and the witness is of the form $A\mathsf{V}$ or $E\mathsf{V}$.
- Otherwise, the play ends in a *tie*.

A *strategy* is a set of rules for a player, telling the player which move to choose in the current configuration. A *winning strategy* from (s, Φ) is a set of rules allowing the player to win every play that starts at (s, Φ) if he plays by the rules. It was shown in [24,25] that the model checking problem of evaluating $[\mathcal{M}, s \models^3 \Phi]$ can be reduced to the problem of finding which player has a winning strategy from (s, Φ) (i.e. to solving the given 3-valued model checking game).

The algorithm proposed in [24,25] for solving the given 3-valued model checking game consists of two parts. First, it constructs a *game-graph*, then it runs an *algorithm for coloring* the game-graph. The game-graph is $G_{\mathcal{M}\times\Phi} = (N, E)$ where $N \subseteq S \times \mathit{sub}(\Phi)$ is the set of nodes and $E \subseteq N \times N$ is the set of edges. N contains a node for each configuration that was reached during the construction of the game-graph that starts from initial configurations $I \times \{\Phi\}$ in a BFS manner, and E contains an edge for each possible move that was applied. The nodes of the game-graph can be classified as: terminal nodes, $\wedge$-nodes, $\vee$-nodes, $A\bigcirc$-nodes, and $E\bigcirc$-nodes. Similarly, the edges can be classified as: progress edges, which originate in $A\bigcirc$ or $E\bigcirc$ nodes and reflect real transitions of the MTS $\mathcal{M}$, and auxiliary nodes, which are all other edges. We distinguish two types of progress edges, two types of children, and two types of SCCs (Strongly Connected Components). *Must-edges* (*may-edges*) are edges based on must-transitions (may-transitions) of MTSs. A node n' is a *must-child* (*may-child*) of the node n if there exists a must-edge (may-edge) (n, n'). A *must-SCC* (*may-SCC*) is an SCC in which all progress edges are must-edges (may-edges).

The game-graph is partitioned into its may-Maximal SCCs (may-MSCCs), denoted Q_i's. This partition induces a partial order $\leq$ on the Q_i's, such that edges go out of a set Q_i only to itself or to a smaller set Q_j. The partial order is extended to a total order $\leq$ arbitrarily. The *coloring algorithm* processes the Q_i's according to $\leq$, bottom-up. Let Q_i be the smallest set that is not fully colored. The nodes of Q_i are colored in two phases, as follows.
Phase 1. Apply these rules to all nodes in Q_i until none of them is applicable.

- A terminal node C is colored: by T if Player $\exists$ wins in it (when $C = (s, \mathit{true})$ or $C = (s, a)$ with $a \in L(s)$ or $C = (s, \neg a)$ with $a \notin L(s)$); and by F if Player $\forall$ wins in it (when $C = (s, \mathit{false})$ or $C = (s, a)$ with $a \notin L(s)$ or $C = (s, \neg a)$ with $a \in L(s)$).

– An $A\bigcirc$ node is colored: by T if all its may-children are colored by T; by F if it has a must-child colored by F; by ? if all its must-children are colored by T or ?, and it has a may-child colored by F or ?.
– An $E\bigcirc$ node is colored: by T if it has a must-child colored by T; by F if all its may-children are colored by F; by ? if it has a may-child colored by T or ?, and all its must-children are colored by F or ?.
– An $\wedge$-node ($\vee$-node) is colored: by T (F) if both its children are colored by T (F); by F (T) if it has a child that is colored by F (T); by ? if it has a child colored by ? and the other child is colored by ? or T (F).

Phase 2. If after propagation of the rules of Phase 1, there are still nodes in Q_i that remain uncolored, then Q_i must be a non-trivial may-MSCC that has exactly one witness. We consider two cases.

Case U. The witness is of the form $A(\Phi_1 \mathsf{U} \Phi_2)$ or $E(\Phi_1 \mathsf{U} \Phi_2)$.
Phase 2a. Repeatedly color by ? each node in Q_i that satisfies one of the following conditions, until there is no change:
(1) An $A\bigcirc$ node that all its must-children are colored by T or ?; (2) An $E\bigcirc$ node that has a may-child colored by T or ?; (3) An $\wedge$ node that both its children are colored T or ?; (4) An $\vee$ node that has a child colored by T or ?. In fact, each node for which the F option is no longer possible according to the rules of Phase 1 is colored by ?.
Phase 2b. Color the remaining nodes in Q_i by F.
Case V. The witness is of the form $A(\Phi_1 \mathsf{V} \Phi_2)$ or $E(\Phi_1 \mathsf{V} \Phi_2)$ (see [16, Appendix B]).

The result of the coloring is a *3-valued coloring function* $\chi : N \to \{T, F, ?\}$.

Theorem 2 ([24]). *For each $n = (s, \Phi') \in G_{\mathcal{M}\times\Phi}$:*

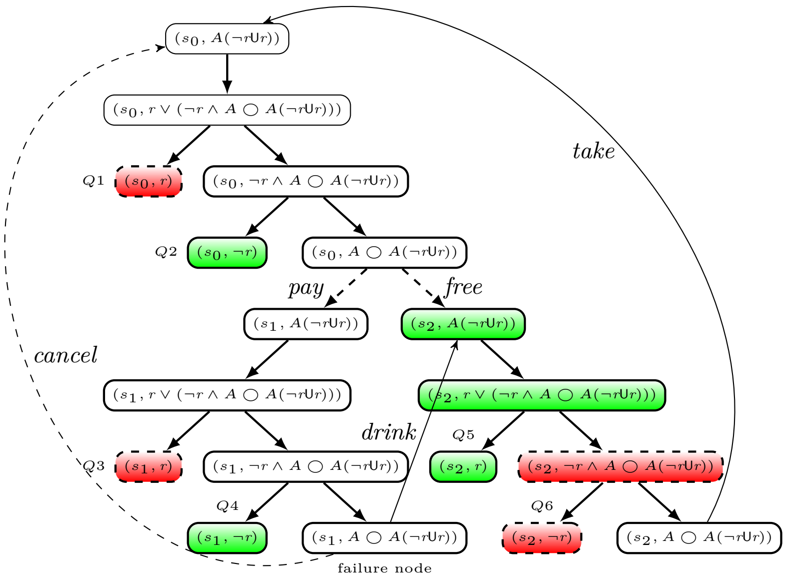
(1) $[(\mathcal{M}, s) \models^3 \Phi'] = tt$ *iff* $\chi(n) = T$ *iff Player* $\exists$ *has a winning strategy at* n*.*
(2) $[(\mathcal{M}, s) \models^3 \Phi'] = f\!f$ *iff* $\chi(n) = F$ *iff Player* $\forall$ *has a winning strategy at* n*.*
(3) $[(\mathcal{M}, s) \models^3 \Phi'] = \bot$ *iff* $\chi(n) = ?$ *iff none of players has a winning strategy at* n*.*

Using Theorem 1 and Theorem 2, given the colored game-graph of the MTS $\boldsymbol{\alpha}^{\text{join}}(\mathcal{F})$, if all its initial nodes are colored by T then $[\mathcal{F} \models \Phi] = tt$, if at least one of them is colored by F then $[\mathcal{F} \models \Phi] = f\!f$. Otherwise, we do not know.

Example 4. The colored game-graph for the MTS $\boldsymbol{\alpha}^{\text{join}}(\textsc{VendMach})$ and $\Phi_1 = A(\neg r \mathsf{U} r)$ is shown in Fig. 4. Green, red (with dashed borders), and white nodes denote nodes colored by T, F, and ?, respectively. The partitions from Q_1 to Q_6 consist of a single node shown in Fig. 4, while Q_7 contains all the other nodes. The initial node (s_0, Φ_1) is colored by ?, so we obtain an indefinite answer. □

5 Incremental Refinement Framework

Given an FTS $\pi_{\mathbb{K}'}(\mathcal{F})$ with a configuration set $\mathbb{K}' \subseteq \mathbb{K}$, we show how to exploit the game-graph of the abstract MTS $\mathcal{M} = \boldsymbol{\alpha}^{\text{join}}(\pi_{\mathbb{K}'}(\mathcal{F}))$ in order to do refinement



$(s_0, A(\neg r \mathsf{U} r))$
$(s_0, r \vee (\neg r \wedge A \bigcirc A(\neg r \mathsf{U} r)))$
$Q1$
(s_0, r)
$(s_0, \neg r \wedge A \bigcirc A(\neg r \mathsf{U} r))$
$Q2$
$(s_0, \neg r)$
$(s_0, A \bigcirc A(\neg r \mathsf{U} r))$
pay
free
take
$(s_1, A(\neg r \mathsf{U} r))$
$(s_2, A(\neg r \mathsf{U} r))$
cancel
$(s_1, r \vee (\neg r \wedge A \bigcirc A(\neg r \mathsf{U} r)))$
$(s_2, r \vee (\neg r \wedge A \bigcirc A(\neg r \mathsf{U} r)))$
drink
$Q5$
$Q3$
(s_1, r)
$(s_1, \neg r \wedge A \bigcirc A(\neg r \mathsf{U} r))$
(s_2, r)
$(s_2, \neg r \wedge A \bigcirc A(\neg r \mathsf{U} r))$
$Q4$
$Q6$
$(s_1, \neg r)$
$(s_1, A \bigcirc A(\neg r \mathsf{U} r))$
$(s_2, \neg r)$
$(s_2, A \bigcirc A(\neg r \mathsf{U} r))$
failure node

Fig. 4: The colored game-graph for $\boldsymbol{\alpha}^{\text{join}}(\textsc{VendMach})$ and $\Phi_1 = A(\neg r \mathsf{U} r)$.

in case that the model checking resulted in an indefinite answer. The refinement consists of two parts. First, we use the information gained by the coloring algorithm of $G_{\mathcal{M}\times\Phi}$ in order to split the single abstract configuration $\mathit{true} \in \boldsymbol{\alpha}^{\text{join}}(\mathbb{K}')$ that represents the whole concrete configuration set $\mathbb{K}'$. We then construct the refined abstract models, using the refined abstract configurations.

There are a failure node and a failure reason associated with an indefinite answer. The goal in the refinement is to find and eliminate at least one of the failure reasons.

Definition 5. *A node n is a* failure node *if it is colored by $?$, whereas none of its children was colored by $?$ at the time n got colored by the coloring algorithm.*

Such failure node can be seen as the point where the loss of information occurred, so we can use it in the refinement step to change the final model checking result.

Lemma 1 ([24]). *A failure node is one of the following.*

- *An $A\bigcirc$-node ($E\bigcirc$-node) that has a may-child colored by F (T).*
- *An $A\bigcirc$-node ($E\bigcirc$-node) that was colored during Phase 2a based on an AU (AV) witness, and has a may-child colored by $?$.*

Given a failure node $n = (s, \Phi)$, suppose that its may-child is $n' = (s', \Phi'_1)$ as identified in Lemma 1. Then the may-edge from n to n' is considered as

```
Algorithm. Verify($\mathcal{F}, \mathbb{K}, \Phi$)

1 Check by game-based model checking algorithm $[\boldsymbol{\alpha}^{\text{join}}(\mathcal{F}) \models^3 \Phi]$?
2 If the result is $tt$, then return that $\Phi$ is satisfied for all variants in $\mathbb{K}$. If the result
  is $f\!f$, then return that $\Phi$ is violated for all variants in $\mathbb{K}$.
3 Otherwise, an indefinite result is returned. Let the may-edge from $n = (s, \Phi_1)$ to
  $n' = (s', \Phi'_1)$ be the reason for failure, and let $\psi$ be the feature expression guarding
  the transition from $s$ to $s'$ in $\mathcal{F}$. We generate $\mathcal{F}_1 = \pi_{[\![\psi]\!]}(\mathcal{F})$ and $\mathcal{F}_2 = \pi_{[\![\neg\psi]\!]}(\mathcal{F})$,
  and call Verify($\mathcal{F}_1, \mathbb{K} \cap [\![\psi]\!], \Phi$) and Verify($\mathcal{F}_2, \mathbb{K} \cap [\![\neg\psi]\!], \Phi$).
```

Fig. 5: The Refinement Procedure that checks $[\mathcal{F} \models \Phi]$.

the failure reason. Since the failure reason is a may-transition in the abstract MTS $\boldsymbol{\alpha}^{\text{join}}(\pi_{\mathbb{K}'}(\mathcal{F}))$, it needs to be refined in order to result either in a must transition or no transition at all. Let $s \xrightarrow{\alpha/\psi} s'$ be the transition in the concrete model $\pi_{\mathbb{K}'}(\mathcal{F})$ corresponding to the above (failure) may-transition. We split the configuration space $\mathbb{K}'$ into $[\![\psi]\!]$ and $[\![\neg\psi]\!]$ subsets, and we partition $\pi_{\mathbb{K}'}(\mathcal{F})$ in $\pi_{[\![\psi]\!]\cap\mathbb{K}'}(\mathcal{F})$ and $\pi_{[\![\neg\psi]\!]\cap\mathbb{K}'}(\mathcal{F})$. Then, we repeat the verification process based on abstract models $\boldsymbol{\alpha}^{\text{join}}(\pi_{[\![\psi]\!]\cap\mathbb{K}'}(\mathcal{F}))$ and $\boldsymbol{\alpha}^{\text{join}}(\pi_{[\![\neg\psi]\!]\cap\mathbb{K}'}(\mathcal{F}))$. Note that, in the former, $\boldsymbol{\alpha}^{\text{join}}(\pi_{[\![\psi]\!]\cap\mathbb{K}'}(\mathcal{F}))$, $s \xrightarrow{\alpha} s'$ becomes a must-transition, while in the latter, $\boldsymbol{\alpha}^{\text{join}}(\pi_{[\![\neg\psi]\!]\cap\mathbb{K}'}(\mathcal{F}))$, $s \xrightarrow{\alpha} s'$ is removed. The complete refinement procedure is shown in Fig. 5. We prove that (see [16, Appendix A]):

Theorem 3. *The procedure Verify($\mathcal{F}, \mathbb{K}, \Phi$) terminates and is correct.*

Example 5. We can do a failure analysis on the game-graph of $\boldsymbol{\alpha}^{\text{join}}(\text{VendMach})$ in Fig. 4. The failure node is $(s_1, A \bigcirc A(\neg r \mathsf{U} r))$ and the reason is the may-edge $(s_1, A \bigcirc A(\neg r \mathsf{U} r)) \xrightarrow{cancel} (s_0, A(\neg r \mathsf{U} r))$. The corresponding concrete transition in VendMach is $s_1 \xrightarrow{cancel/c} s_0$. So, we partition the configuration space $\mathbb{K}^{\text{VM}}$ into subsets $[\![c]\!]$ and $[\![\neg c]\!]$, and in the next second iteration we consider FTSs $\pi_{[\![c]\!]}(\text{VendMach})$ and $\pi_{[\![\neg c]\!]}(\text{VendMach})$. $\square$

The game-based model checking algorithm provides us with a convenient framework to use results from previous iterations and avoid unnecessary calculations. At the end of the i-th iteration of abstraction-refinement, we remember those nodes that were colored by definite colors. Let D denote the set of such nodes. Let $\chi_D : D \to \{T, F\}$ be the coloring function that maps each node in D to its definite color. The incremental approach uses this information both in the construction of the game-graph and its coloring. During the construction of a new refined game-graph performed in a BFS manner in the next $i + 1$-th iteration, we prune the game-graph in nodes that are from D. When a node $n \in D$ is encountered, we add n to the game-graph and do not continue to construct the game-graph from n onwards. That is, $n \in D$ is considered as terminal node and colored by its previous color. As a result of this pruning, only the reachable sub-graph that was previously colored by ? is refined.

Example 6. The property Φ_1 holds for $\pi_{[\![\neg c]\!]}(\text{VendMach})$. The initial node of the game-graph $G_{\boldsymbol{\alpha}^{\text{join}}(\pi_{[\![\neg c]\!]}(\text{VendMach}))\times\Phi_1}$ (see [16, Fig. 13, Appendix C]), is colored

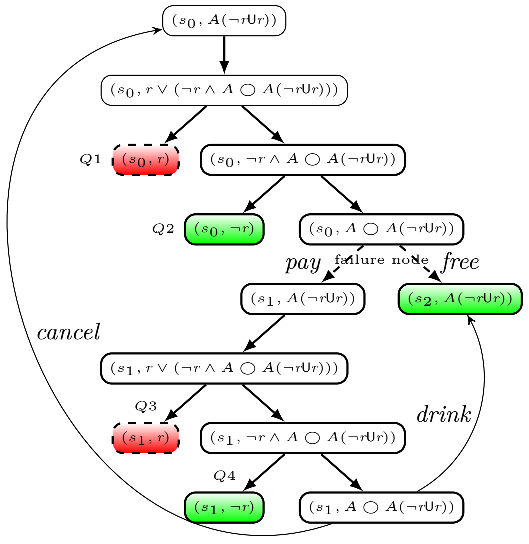

$(s_0, A(\neg r \mathsf{U} r))$
$(s_0, r \vee (\neg r \wedge A \bigcirc A(\neg r \mathsf{U} r)))$
Q1
(s_0, r)
$(s_0, \neg r \wedge A \bigcirc A(\neg r \mathsf{U} r))$
Q2
$(s_0, \neg r)$
$(s_0, A \bigcirc A(\neg r \mathsf{U} r))$
failure node
pay
free
$(s_1, A(\neg r \mathsf{U} r))$
$(s_2, A(\neg r \mathsf{U} r))$
cancel
$(s_1, r \vee (\neg r \wedge A \bigcirc A(\neg r \mathsf{U} r)))$
Q3
(s_1, r)
$(s_1, \neg r \wedge A \bigcirc A(\neg r \mathsf{U} r))$
drink
Q4
$(s_1, \neg r)$
$(s_1, A \bigcirc A(\neg r \mathsf{U} r))$

Fig. 6: $G_{\boldsymbol{\alpha}^{\text{join}}(\pi_{[\![c]\!]}(\textsc{VendMach}))\times\Phi_1}$.

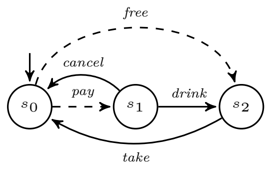


Fig. 7: $\boldsymbol{\alpha}^{\text{join}}(\pi_{[\![c]\!]}(\textsc{VendMach}))$

by T. On the other hand, we obtain an indefinite answer for $\pi_{[\![c]\!]}(\textsc{VendMach})$. The model $\boldsymbol{\alpha}^{\text{join}}(\pi_{[\![c]\!]}(\textsc{VendMach}))$ is shown in Fig. 7, whereas the final colored game-graph $G_{\boldsymbol{\alpha}^{\text{join}}(\pi_{[\![c]\!]}(\textsc{VendMach}))\times\Phi_1}$ is given in Fig. 6. The failure node is $(s_0, A\bigcirc A(\neg r \mathsf{U} r))$, and the reason is the may-edge $(s_0, A\bigcirc A(\neg r \mathsf{U} r)) \xrightarrow{pay} (s_1, A(\neg r \mathsf{U} r))$. The corresponding concrete transition in $\pi_{[\![c]\!]}(\textsc{VendMach})$ is $s_0 \xrightarrow{pay/\neg f} s_1$. So, in the next third iteration we consider FTSs $\pi_{[\![c\wedge\neg f]\!]}(\textsc{VendMach})$ and $\pi_{[\![c\wedge f]\!]}(\textsc{VendMach})$.

The initial node of the graph $G_{\boldsymbol{\alpha}^{\text{join}}(\pi_{[\![c\wedge\neg f]\!]}(\textsc{VendMach}))\times\Phi_1}$ (see [16, Fig. 16, Appendix C]) is colored by F in Phase 2b. The initial node of $G_{\boldsymbol{\alpha}^{\text{join}}(\pi_{[\![c\wedge f]\!]}(\textsc{VendMach}))\times\Phi_1}$ (see [16, Fig. 17, Appendix C]) is colored by T.

In the end, we conclude that Φ_1 is satisfied by the variants $\{\neg c \wedge \neg f, \neg c \wedge f, c \wedge f\}$, and Φ is violated by the variant $\{c \wedge \neg f\}$.

On the other hand, we need two iterations to conclude that $\Phi_2 = E(\neg r \mathsf{U} r)$ is satisfied by all variants in $\mathbb{K}^{\text{VM}}$ (see [16, Appendix D] for details). $\square$

6 Evaluation

To evaluate our approach, we use a synthetic example to demonstrate specific characteristics of our approach, and the Elevator model which is often used as benchmark in SPL community [23,4,15,12,20]. We compare (1) our abstraction-refinement procedure `Verify` with the game-based model checking algorithm implemented in Java from scratch vs. (2) family-based version of the NuSMVmodel checker, denoted fNuSMV, which implements the standard lifted model checking algorithm [5]. For each experiment, we measure T(ime) to perform an analysis task, and Call which is the number of times an approach calls the model checking

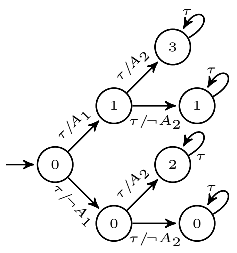


Fig. 8: The model M_2.

n	Φ				Φ'			
	fNuSMV		`Verify`		fNuSMV		`Verify`	
	Call	T	Call	T	Call	T	Call	T
2	1	0.08	1	0.07	1	0.08	5	0.83
7	1	1.64	1	0.16	1	1.68	15	2.68
10	1	992.80	1	0.68	1	1019.27	21	4.57
11	1	infeasible	1	1.42	1	infeasible	23	5.98
15	1	infeasible	1	26.55	1	infeasible	31	41.64

Fig. 9: Verification of M_n (T in seconds).

engine. All experiments were executed on a 64-bit Intel®Core^{TM} i5-3337U CPU running at 1.80 GHz with 8 GB memory. All experimental data is available from: `https://aleksdimovski.github.io/automatic-ctl.html`.

Synthetic example. The FTS M_n (where $n > 0$) consists of n features $A_1, \ldots, A_n$ and an integer data variable x, such that the set AP consists of all evaluations of x which assign nonnegative integer values to x. The set of valid configurations is $\mathbb{K}_n = 2^{\{A_1,\ldots,A_n\}}$. M_n has a tree-like structure, where in the root is the initial state with $x = 0$. In each level k ($k \geq 1$), there are two states that can be reached with two transitions leading from a state from a previous level. One transition is allowable for variants with the feature A_k enabled, so that in the target state the variable's value is $x + 2^{k-1}$ where x is its value in the source state, whereas the other transition is allowable for variants with A_k disabled, so that the value of x does not change. For example, M_2 is shown in Fig. 8, where in each state we show the current value of x and all transitions have the silent action τ.

We consider two properties: $\Phi = A(\mathit{true}\,\mathsf{U}(x \geq 0))$ and $\Phi' = A(\mathit{true}\,\mathsf{U}(x \geq 1))$. The property Φ is satisfied by all variants in $\mathbb{K}$, whereas Φ' is violated only by one configuration $\neg A_1 \wedge \ldots \wedge \neg A_n$ (where all features are disabled). We have verified M_n against Φ and Φ' using fNuSMV(e.g. see fNuSMVmodels for M_1 and M_2 in [16, Fig. 23, Appendix E]). We have also checked M_n using our `Verify` procedure. For Φ, `Verify` terminates in one iteration since $\boldsymbol{\alpha}^{\text{join}}(M_n)$ satisfies Φ (see $G_{\boldsymbol{\alpha}^{\text{join}}(M_1)\times\Phi}$ in [16, Fig. 24, Appendix E]). For Φ', `Verify` needs $n + 1$ iterations. First, an indefinite result is reported for $\boldsymbol{\alpha}^{\text{join}}(M_n)$ (e.g. see $G_{\boldsymbol{\alpha}^{\text{join}}(M_1)\times\Phi'}$ in [16, Fig. 27, Appendix E]), and the configuration space is split into $[\![\neg A_1]\!]$ and $[\![A_1]\!]$ subsets. The refinement procedure proceeds in this way until we obtain definite results for all variants. The performance results are shown in Fig. 9. Notice that, fNuSMV reports all results in only one iteration. As n grows, `Verify` becomes faster than fNuSMV. For $n = 11$ ($|\mathbb{K}| = 2^{11}$), fNuSMV timeouts after 2 hours. In contrast, `Verify` is feasible even for large values of n.

Elevator. We have experimented with the Elevator model with four floors, designed by Plath and Ryan [23]. It contains about 300 LOC of fNuSMV code and 9 independent optional features that modify the basic behaviour of the elevator, thus yielding $2^9 = 512$ variants. To use our `Verify` procedure, we have manually translated the fNuSMV model into an FTS and then we

prop- *-erty*	fNuSMV		`Verify`		*Improvement*
	CALL	T	CALL	T	TIME
Φ_1	1	15.22 s	1	0.55 s	28 ×
Φ_2	1	1.59 s	1	0.59 s	2.7 ×
Φ_3	1	1.76 s	1	0.67 s	2.6 ×

Fig. 10: Verification of ELEVATOR properties (T in seconds).

have called `Verify` on it. The basic ELEVATOR system consists of a single lift that travels between four floors. There are four platform buttons and a single lift, which declares variables $floor, door, direction$, and a further four cabin buttons. When serving a floor, the lift door opens and closes again. We consider three properties "$\Phi_1 = E(tt\,\mathsf{U}(floor = 1 \land idle \land door = closed))$", "$\Phi_2 = A(tt\,\mathsf{U}(floor = 1 \land idle \land door = closed))$", and "$\Phi_3 = E(tt\,\mathsf{U}((floor = 3 \land \neg liftBut3.pressed \land direction = up) \implies door = closed))$". The performance results are shown in Fig. 10. The properties Φ_1 and Φ_2 are satisfied by all variants, so `Verify` achieves speed-ups of 28 times for Φ_1 and 2.7 times for Φ_2 compared to the fNuSMV approach. fNuSMV takes 1.76 sec to check Φ_3, whereas `Verify` ends in 0.67 sec thus giving 2.6 times performance speed-up.

7 Related Work and Conclusion

There are different formalisms for representing variability models [21,2]. Classen et al. [4] present Featured Transition Systems (FTSs). They show how specifically designed lifted model checking algorithms [7,5] can be used for verifying FTSs against LTL and CTL properties. The variability abstractions that preserve LTL are introduced in [14,15,17], and subsequently automatic abstraction refinement procedures [8,18] for lifted model checking of LTL are proposed, by using Craig interpolation to define the refinement. The variability abstractions that preserve the full CTL are introduced in [12], but they are constructed manually and no notion of refinement is defined there. In this paper, we define an automatic abstraction refinement procedure for lifted model checking of full CTL by using games to define the refinement. To the best of our knowledge, this is the first such procedure in lifted model checking.

One of the earliest attempts for using games for CTL model checking has been proposed by Stirling [26]. Shoham and Grumberg [24,25,19,3] have extended this game-based approach for CTL over 3-valued semantics. In this work, we exploit and apply the game-based approach in a completely new direction, for automatic CTL verification of variability models.

The works [11,13] present an approach for software lifted model checking of `#ifdef`-based program families using symbolic game semantics models [10].

To conclude, in this work we present a game-based lifted model checking for abstract variability models with respect to the full CTL. We also suggest an automatic refinement procedure, in case the model checking result is indefinite.

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